

Kley 13/14

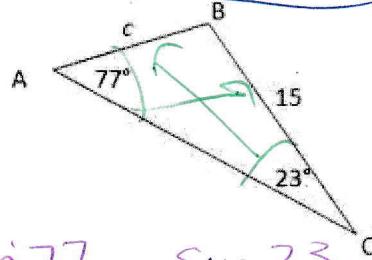
6.1/6.2 Law of Sine and Law of Cosine

*Law of Sine and Law of Cosine worksheets

Examples: Round to four decimal places.

1) Find c .

measure of $\angle Q$.

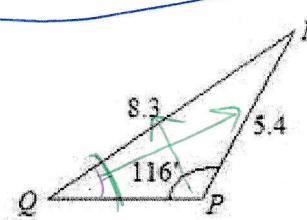


$$\frac{\sin 77}{15} = \frac{\sin 23}{c}$$

$$c = \frac{15 \sin 23}{\sin 77}$$

$$c = 6.0151$$

2) Find the



$$\frac{\sin 116}{8.3} = \frac{\sin Q}{5.4}$$

$$\sin Q = \frac{5.4 \sin 116}{8.3}$$

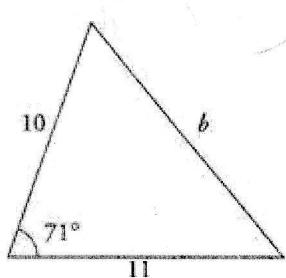
$$\sin^{-1} \sin Q = \sin^{-1} 0.58 \dots$$

$$Q = 35.7859$$

Examples: Round to four decimal places.

3) Find b .

4) Find $m\angle A$.

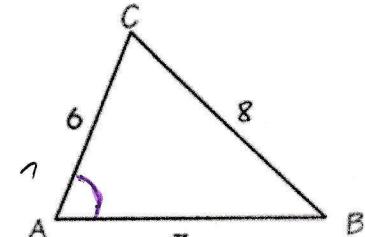


$$b^2 = 10^2 + 11^2 - 2(10)(11)\cos 71$$

$$b^2 = 221 - 220 \cos 71$$

$$\sqrt{b^2} = \sqrt{49.375006}$$

$$b = 12.2219$$



$$8^2 = 6^2 + 7^2 - 2(6)(7)\cos A$$

$$64 = 85 - 84 \cos A$$

$$-21 = -84 \cos A$$

$$\cos^{-1} \frac{1}{4} = \cos A \cos^{-1}$$

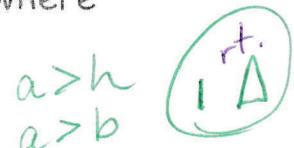
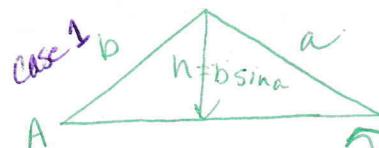
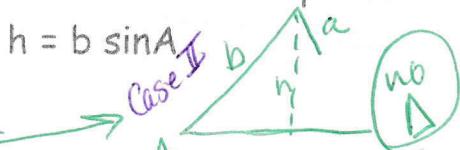
$$A = 75.5225^\circ$$

6.1/6.2 The Ambiguous Case and Applications

Ambiguous Case of the Law of Sines (SSA)

*The given information may result in one triangle, two triangles, or no triangle at all

*The number of possible triangles, if any, that can be formed in the SSA case depends on h (the length of the altitude) where



Case 1) a is just right, we have one right triangle

Case 2) a is too small, we don't have a triangle at all

Case 3) a is too big

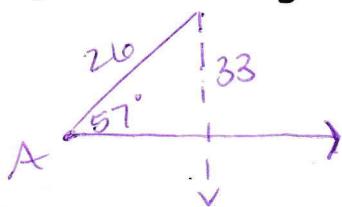
➤ and a is bigger than b , we have one triangle

(pendulum away from A)

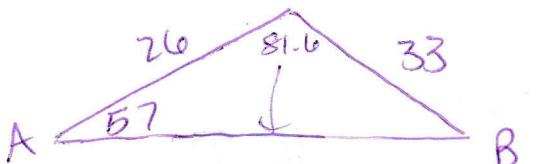
➤ and a is smaller than b , we have two triangles
(pendulum in and out from A)

One Solution:

1) Solve triangle ABC



(redraw)



$$\frac{\sin 81.6}{c} = \frac{\sin 57}{33}$$

$$c = 38.926$$

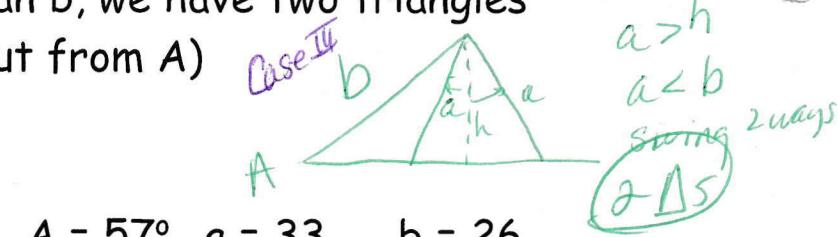
$$\frac{\sin 57}{33} = \frac{\sin B}{26}$$

$$\frac{26 \sin 57}{33} = \sin B$$

$$\sin^{-1}\left(\frac{26 \sin 57}{33}\right)$$

$$\begin{aligned} \angle C &= 180 - 57 - 41.4 \\ \angle C &= 81.6^\circ \end{aligned}$$

$$\begin{aligned} \angle B &= 41.4^\circ \end{aligned}$$



$$\begin{aligned} A &= 57^\circ & a &= 33 & b &= 26 \\ \text{opp} & & & & & \text{side} \end{aligned}$$

$$\sin 57 = \frac{33}{26}$$

$$26 \sin 57 = 33$$

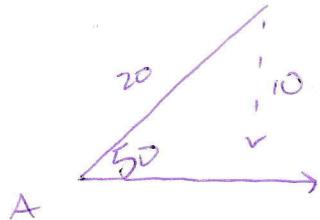
$$21.808 < 33$$

- * cant be right
- * cant be 2 Δ's

a is too big to be right

No Solution:

2) Solve triangle ABC $A = 50^\circ$ $a = 10$ $b = 20$



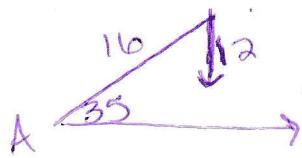
$$\frac{\sin 50}{10} = \frac{\sin B}{20}$$
$$20 \sin 50 = \sin B$$

$$1.532 = \sin B \leftarrow \sin \text{ cannot exceed } 1$$

\therefore There is no Δ with the given measurements.

Two Solutions:

3) Solve triangle ABC $A = 35^\circ$ $a = 12$ $b = 16$



* use the sin test

$$\sin 35 = \frac{12}{16}$$

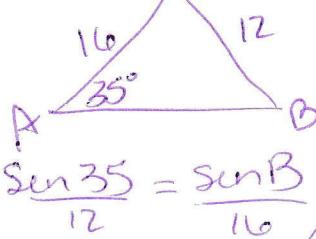
$$16 \sin 35 = 12$$

$$9.177 < 12$$

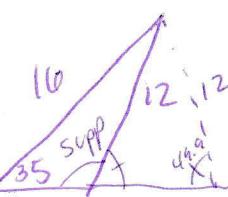
$a > b$ but bigger than
 $a > h$ not rt Δ

$a > h$
 a is too big $\because a > b$ so
we have 2 Δ 's

* Redraw w/ 2 Δ 's



$$\frac{\sin 35}{12} = \frac{\sin B}{16}$$



* the Δ 's will be supplementary

$$\therefore B = 180 - 49.886 = 130.1^\circ$$

$$\therefore C = 14.9^\circ$$

$$\frac{\sin 35}{12} = \frac{\sin 14.9}{c}$$

$$\therefore c = 5.379$$

$$\frac{\sin 35}{12} = \frac{\sin 49.9}{b}$$
$$\therefore b = 16$$
$$\therefore C = 95.1^\circ$$

$$\frac{\sin 35}{12} = \frac{\sin 95.1}{c}$$

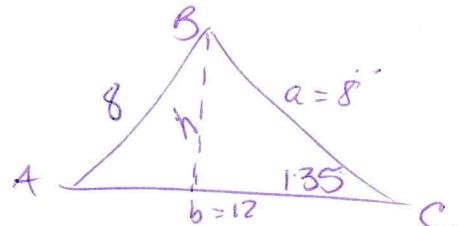
$$\therefore c = 20.8$$

Area of an oblique Triangle

$$\begin{aligned} \text{Area} &= \frac{1}{2} bc \sin A \\ &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} ac \sin B \end{aligned} \quad \left. \begin{array}{l} \text{included} \\ \text{xs} \end{array} \right\}$$

- 4) Find the area of a triangle having two sides of length 8 meters and 12 meters and an included angle of 135° . Round to the nearest square meter.

$$A = \frac{1}{2}(8)(12)\sin 135^\circ$$

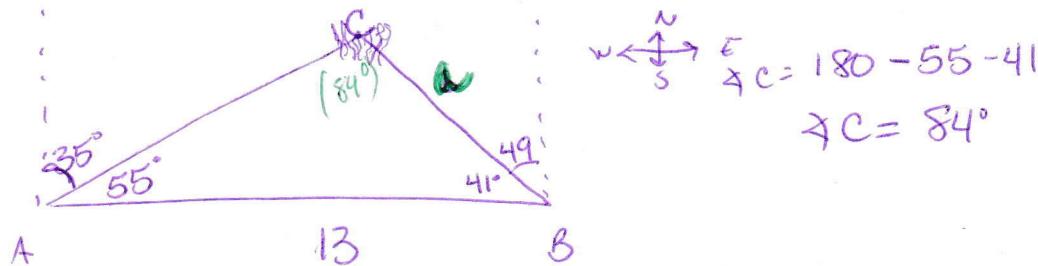


$$A = 33.94$$

$$\boxed{34 \text{ m}^2}$$

Applications

- 5) Two fire-lookout stations are 13 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is N 35° E and the bearing of the fire from station B is N 49° W. How far, to the nearest tenth of a mile is the fire from station B?

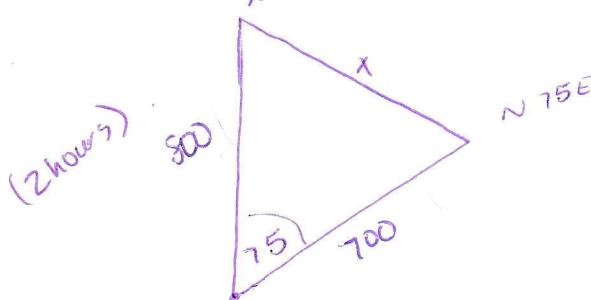


$$\frac{\sin 84}{13} = \frac{\sin 55}{\text{or } a}$$

$$a = \frac{13 \sin 55}{\sin 84}$$

The fire is approximately 10.7 miles from Station B

6) Two airplanes leave an airport at the same time on different runways. One flies directly north at 400 miles per hour. The other airplane flies on a bearing of N 75° E at 350 miles per hour. How far apart will the airplanes be after two hours?



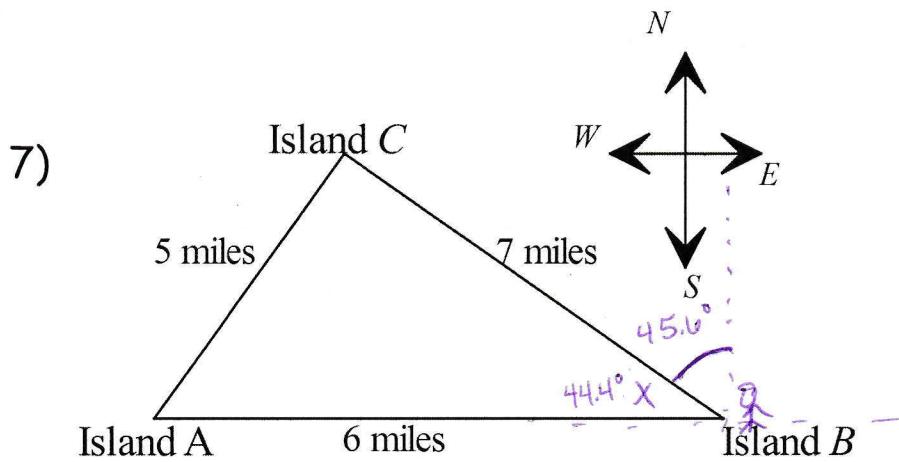
$$x^2 = 800^2 + 700^2 - 2(800)(700)\cos 75$$

$$x^2 = 1130000 - 1120000 \cos 75$$

$$\sqrt{x^2} = \sqrt{840122.6695}$$

$$x = 916.58$$

≈ 917 miles apart



If you are on island B, what bearing should you navigate to go to island C?

$$5^2 = 6^2 + 7^2 - 2(6)(7)\cos X$$

$$25 = 85 - 84 \cos X$$

$$-60 = -84 \cos X$$

$$\frac{-60}{-84} = \cos X \cos^{-1}$$

$$X = 44.4153$$

$$90 - 44.4153^\circ = 45.58^\circ \quad \boxed{N 45.6^\circ W}$$

Heron's Formula:

The area of a triangle with sides a , b and c is

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

where s is one-half its perimeter: $s = \frac{1}{2}(a + b + c)$

- 8) Find the area of the triangle with $a = 6$ meters, $b = 16$ meters, and $c = 18$ meters. Round to the nearest square meters.

$$s = \frac{1}{2}(6 + 16 + 18)$$

$$s = 20$$

$$\begin{aligned} A &= \sqrt{20(20-6)(20-16)(20-18)} \\ &= \sqrt{20(14)(4)(2)} \\ &= \sqrt{2240} \\ &\approx 47 \text{ m}^2 \end{aligned}$$

- 9) Bob Fernando's triangular property has the measurements of 40 yards, 50 yards and 30 yards. Find the area of the property.

$$s = \frac{1}{2}(40 + 50 + 30)$$

$$s = 60$$

$$\begin{aligned} A &= \sqrt{60(60-40)(60-50)(60-30)} \\ &= \sqrt{60(20)(10)(30)} \\ &= \sqrt{360000} \\ &= \boxed{600 \text{ yd}^2} \end{aligned}$$

6.6 Vectors

A vector is a directed line segment

- Magnitude (length) $\|m\|$ - distance of m distance formula
- Direction (angle measure) degrees or radians slope

*For vectors to be equal, they must have the same magnitude and direction.

*Vectors are usually denoted by boldface letters, but can also be written as \vec{v}

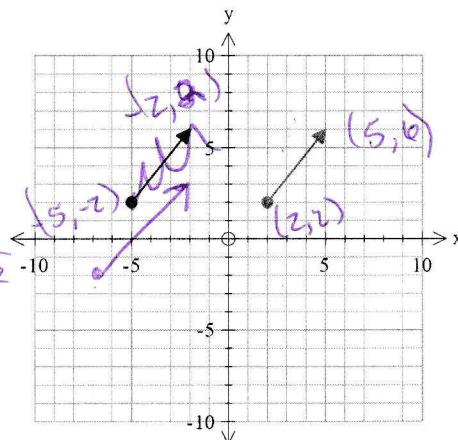
\overrightarrow{PQ} P is the initial point and Q is the terminal point

1) show that $\mathbf{u} = \mathbf{v}$

To find $\|m\| = \sqrt{x^2 + y^2}$
To find direction = $\tan^{-1}\left(\frac{y}{x}\right)$

$$\begin{aligned} \|\mathbf{u}\| &= \sqrt{(-2+5)^2 + (2+2)^2} & \|\mathbf{v}\| &= \sqrt{(3)^2 + (4)^2} \\ &= \sqrt{9+16} & &= \sqrt{9+16} \\ &= \sqrt{25} & &= \sqrt{25} \\ &= 5 & &= 5 \end{aligned}$$

$\therefore \|\mathbf{u}\| = \|\mathbf{v}\|$



Direction

$$m = \frac{2+2}{-2+5} = \frac{4}{3}$$

$$m = \frac{6-2}{5-2} = \frac{4}{3}$$

They have the same magnitude
& same direction

Scalar Multiplication

If k is a real number and v a vector, the vector kv is called a scalar multiple of the vector v.

- Magnitude of $|k|\|v\|$
- Direction
 - o Same if $k > 0$
 - o Opposite if $k < 0$

Adding Vectors

- Add the x's
- Add the y's

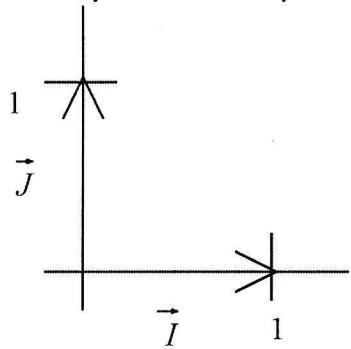
Subtracting Vectors

- Subtract the x's
- Subtract the y's

Unit Vectors

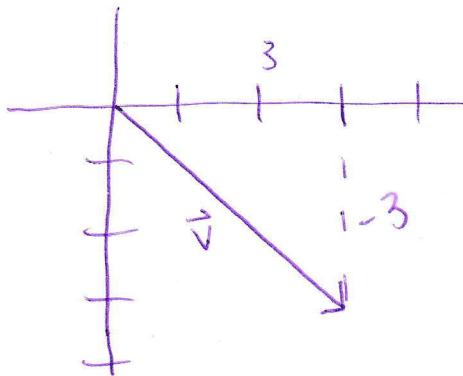
I represents x distance (horizontal component)

J represents y distance (vertical component)



2) Sketch $\vec{v} = 3I - 3J$ and find $\|\vec{v}\|$

> Starts @ (0,0) & ends @ (3, -3)



$$\begin{aligned}a^2 + b^2 &= c^2 \\3^2 + (-3)^2 &= c^2 \\18 &= c^2 \\\vec{v} &= 3\sqrt{2}\end{aligned}$$

3) If $\vec{v} = 7I + 3J$ and $\vec{w} = 4I - 5J$, find $\vec{v} + \vec{w}$ and $\vec{v} - \vec{w}$

$$\vec{v} + \vec{w} = (7I + 3J) + (4I - 5J) = \boxed{11I - 2J}$$

$$\vec{v} - \vec{w} = (7I + 3J) - (4I - 5J) = \boxed{3I + 8J}$$

4) If $\vec{v} = 7I + 10J$, find $8\vec{v}$ and $-5\vec{v}$

$$8\vec{v} = 8(7I + 10J) = \boxed{56I + 80J}$$

$$-5\vec{v} = -5(7I + 10J) = \boxed{-35I - 50J}$$

5) If $v = 7i + 3j$ and $w = 4i - 5j$, find $6v - 3w$

$$6\vec{v} = 6(7i + 3j) = 42i + 18j$$

$$-3\vec{w} = -3(4i - 5j) = -12i + 15j$$

$$\begin{array}{r} \\ + \\ \hline 30i + 33j \end{array}$$

Zero Vector: $\vec{v} = 0I + 0J$

$$\|\vec{v}\| = 0$$

Direction = none

Unit vector in the same direction:

- ✓ • Unit vector has a magnitude of 1 therefore we divide the vector by its own magnitude.
- vector whose magnitude = 1*

$$\frac{\vec{v}}{\|\vec{v}\|}$$



6) Find the unit vector in the same direction of $\vec{v} = 4I - 3J$.

$$\|\vec{v}\| = \sqrt{a^2 + b^2} = \sqrt{4^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$\frac{\vec{v}}{\|\vec{v}\|} = \frac{4i - 3j}{5} = \left[\frac{4}{5}i - \frac{3}{5}j \right]$$

Verify

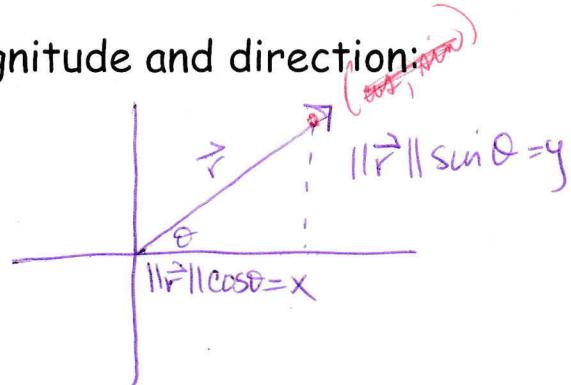
$$\text{Want to } = 1 \quad \sqrt{\left(\frac{4}{5}\right)^2 + \left(-\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25} + \frac{9}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1 \quad \checkmark$$

Writing a vector in terms of its magnitude and direction:

$$\vec{v} = \|\vec{v}\| \cos \theta \cdot I + \|\vec{v}\| \sin \theta \cdot J$$

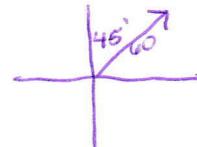
$$\begin{aligned} \cos \theta &= \frac{a}{\|\vec{v}\|} \\ \sin \theta &= \frac{b}{\|\vec{v}\|} \end{aligned}$$

$$\text{so } a = \|\vec{v}\| \cos \theta \quad b = \|\vec{v}\| \sin \theta$$



7) The jet stream is blowing at 60 miles per hour in the direction N 45° E. Express its velocity as a vector \vec{v} in terms of i and j .

$$\|\vec{v}\| = 60$$



$$\begin{aligned} \vec{v} &= \|\vec{v}\| \cos \theta i + \|\vec{v}\| \sin \theta j \\ &= 60 \cos 45^\circ i + 60 \sin 45^\circ j \\ &= 60 \left(\frac{\sqrt{2}}{2} \right) i + 60 \left(\frac{\sqrt{2}}{2} \right) j \end{aligned}$$

$$\boxed{\vec{v} = 30\sqrt{2}i + 30\sqrt{2}j}$$

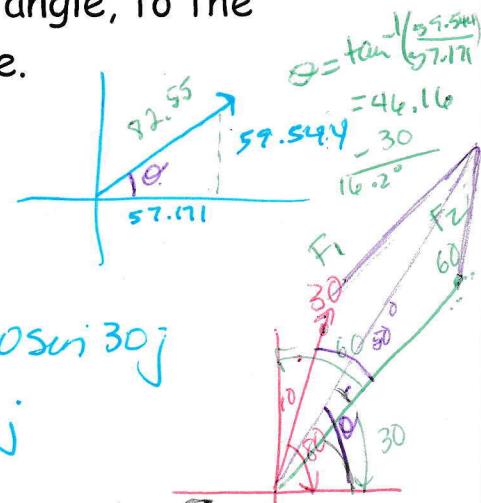
Finding the Resultant Force:

$$\|F_1\| \quad \|F_2\|$$

8) Two forces, F_1 and F_2 , of magnitude 30 and 60 pounds, respectively, act on an object. The direction of F_1 is N 10° E and the direction of F_2 is N 60° E. Find the magnitude, to the nearest hundredth of a pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.

$$F_1 \quad 30 \text{ N } 10^\circ \text{ E} \quad \|F_1\| = 30 \quad \theta = \begin{array}{l} 90 - 10 \\ 80 \\ 90 - 60 \\ 30 \end{array}$$

$$F_2 \quad 60 \text{ N } 60^\circ \text{ E} \quad \|F_2\| = 60 \quad \theta = \begin{array}{l} 57.171 \\ 59.544 \\ 30 \\ 16.2^\circ \end{array}$$



$$F_1 = 30 \cos 80i + 30 \sin 80j$$

$$5.209i + 29.544j$$

$$F_2 = 60 \cos 30i + 60 \sin 30j$$

$$51.962i + 30j$$

$$\text{resultant force : } F_1 + F_2 = 57.171i + 59.544j$$

$$\|F\| = \sqrt{(57.171)^2 + (59.544)^2} = \sqrt{6814.011} \approx 82.547$$

~~$\frac{82.547}{\sin 130^\circ} = \frac{30}{\sin \theta}$~~ ~~$\tan^{-1} \left(\frac{59.544}{57.171} \right) = 46.116^\circ$~~

~~$\theta = 16.2^\circ$~~ $\theta = 46.116^\circ$ $\theta = 30^\circ - 16.2^\circ$

magnitude of the resultant force (1bs)

The two given forces are equivalent to a single force of approx 82.55 pounds w/a direction of $\approx 46.1^\circ$.

6.7 The Dot Product

The dot product of two vectors is the sum of the products of their horizontal components and their vertical components.

vector / addition gives vector
vector must give vector
dot product gives scalar (real #)

$$\text{If } \vec{v} = a_1 i + b_1 j \quad \text{and} \quad \vec{w} = a_2 i + b_2 j$$

$$\text{Then } \vec{v} \cdot \vec{w} = a_1 a_2 + b_1 b_2$$

*This gives us a number, not a vector

*Two vectors are orthogonal if the angle between them is 90°

1) If $\vec{v} = 7\mathbf{i} - 4\mathbf{j}$ and $\vec{w} = 2\mathbf{i} - \mathbf{j}$, find each of the following dot products:

$$\vec{v} \cdot \vec{w} = 7(2) + (-4)(-1)$$

$$\begin{array}{r} 14 \\ + 4 \\ \hline 18 \end{array}$$

$$\vec{w} \cdot \vec{v}$$

$$\boxed{18}$$

$$\vec{w} \cdot \vec{w} = 2(2) + (-1)(-1)$$

$$\begin{array}{r} 4 \\ + 1 \\ \hline 5 \end{array}$$

Properties of the Dot Product if u, v, w are vectors,

1) $\vec{v} \cdot \vec{v} = \vec{v} \cdot \vec{v}$ $v \cdot v = v \cdot v$ $\& c$ is scalar

2) $\vec{v} \vec{v} \vec{v} \vec{v} \vec{v} \vec{v} \vec{v} u(v+w) = uv+uw$

3) $\vec{v} \vec{v} 0 \cdot v = 0$

4) $\vec{v} \vec{v} \vec{v} v \cdot v = \|v\|^2$

5) $\vec{v} \vec{v} \vec{v} \vec{v} \vec{v} (cv) \vec{v} = c(v \cdot v) = c(v)$

Formula for the angle between two vectors:

If v and w are two nonzero vectors and θ is the smallest nonnegative angle between v and w , then

$$\cos \theta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|} \quad \text{and} \quad \theta = \cos^{-1} \left(\frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|} \right)$$

2) Find the angle between the vectors $v = 4i - 3j$ and $w = i + 2j$

Round to the nearest tenth of a degree. $\|v\| = \sqrt{4^2 + 3^2} = 5$
 $\|w\| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$$\cos \theta = \frac{(4)(1) + (-3)(2)}{5(\sqrt{5})}$$

$$= \frac{4 - 6}{5\sqrt{5}}$$

$$\cos^{-1} \cos \theta = \frac{-2\sqrt{5}}{25} \cos^{-1} \approx 100.3^\circ$$

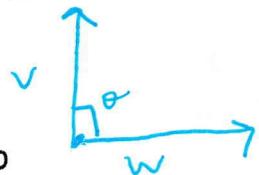
Parallel and Orthogonal Vectors:

- Two vectors are parallel when the angle between the vectors is 0° or 180°



- Two vectors are orthogonal when the angle between the vectors is 90°

*The word orthogonal is used rather than perpendicular to describe vectors that meet at right angles.



- If the dot product is 0, the vectors are orthogonal

$\vec{v} \cdot \vec{w} = 0$, then \vec{v} and \vec{w} are orthogonal \perp
 (opp reciprocal "slope")

3) Are the vectors $\vec{v} = 2i + 3j$ and $\vec{w} = 6i - 4j$ orthogonal?

$$\begin{aligned} \vec{v} \cdot \vec{w} &= 2(6) + (3)(-4) \\ &= 12 - 12 \\ &= 0 \end{aligned}$$

Since $\vec{v} \cdot \vec{w} = 0$, then the vectors
are orthogonal

The Vector Projection of v onto w :

If v and w are two nonzero vectors, the vector projection of v onto w is:

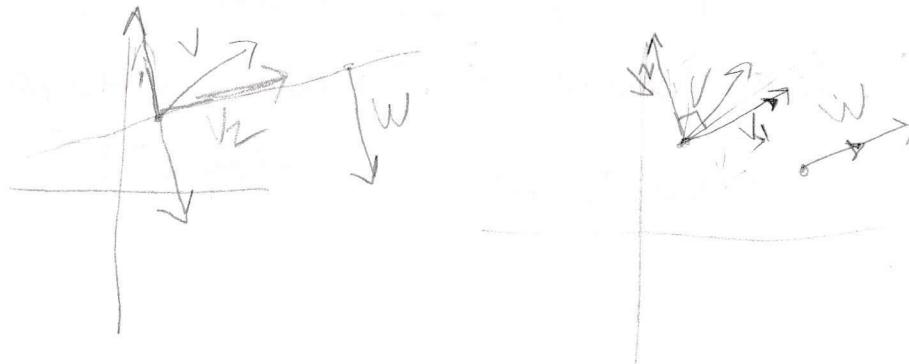
$$V_1 = \text{Proj}_{w}v = \frac{\vec{v} \cdot \vec{w}}{\|\vec{w}\|^2} \cdot \vec{w}, \quad V_2 = v - v_1$$

This tells us how much force is applied in a particular direction

v and w are two nonzero vectors. Vector v can be expressed as the sum of two orthogonal vectors v_1 and v_2 , where v_1 is parallel to w and v_2 is orthogonal to w .

The vectors v_1 and v_2 are called the **vector components** of v .

The process of expressing v as v_1+v_2 is called the **decomposition** of v into v_1 and v_2



4) If $\mathbf{v} = 2\mathbf{i} - 5\mathbf{j}$ and $\mathbf{w} = \mathbf{i} - \mathbf{j}$, find the vector projection of \mathbf{v} onto \mathbf{w} . $\|\mathbf{w}\| = \sqrt{2}$

$$\text{proj}_{\mathbf{w}} \mathbf{v} = \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{w}\|^2} \mathbf{w} = \frac{(2)(1) + (-5)(-1)}{(\sqrt{2})^2} \mathbf{w}$$

$$= \frac{7}{2} (\mathbf{i} - \mathbf{j}) = \boxed{\frac{7}{2} \mathbf{i} - \frac{7}{2} \mathbf{j}}$$

$$\sqrt{1^2 + (-1)^2} = \sqrt{2}$$

5) Let $\mathbf{v} = 2\mathbf{i} - 5\mathbf{j}$ and $\mathbf{w} = \mathbf{i} - \mathbf{j}$. Decompose \mathbf{v} into two vectors \mathbf{v}_1 and \mathbf{v}_2 where \mathbf{v}_1 is parallel to \mathbf{w} and \mathbf{v}_2 is orthogonal to \mathbf{w} .

$$\mathbf{v}_1 = \text{proj}_{\mathbf{w}} \mathbf{v} = \frac{7}{2} \mathbf{i} - \frac{7}{2} \mathbf{j} \quad (\text{from previous example})$$

$$\begin{aligned} \mathbf{v}_2 &= \mathbf{v} - \mathbf{v}_1 = (2\mathbf{i} - 5\mathbf{j}) - \left(\frac{7}{2} \mathbf{i} - \frac{7}{2} \mathbf{j} \right) \\ &= \frac{4}{2} \mathbf{i} - \frac{10}{2} \mathbf{j} - \frac{7}{2} \mathbf{i} + \frac{7}{2} \mathbf{j} \end{aligned}$$

$$\boxed{-\frac{3}{2} \mathbf{i} - \frac{3}{2} \mathbf{j}}$$

*mag of force
distance over which
constant force is
applied*

Def of work: $W = \mathbf{F} \cdot \overrightarrow{AB} = \|\mathbf{F}\| \|\overrightarrow{AB}\| \cos \theta$ ** between force & direction of motion*

6) A child pulls a wagon along level ground by exerting a force of 20 lbs on a handle that makes an angle of 30° with the horizontal. How much work is done pulling the wagon 150 ft?

$$W = \|\mathbf{F}\| \|\overrightarrow{AB}\| \cos \theta$$

$$= (20)(150)\cos 30$$

$$= 2598.07$$

x 2598 foot pounds

(7) Force is given by $\mathbf{F} = 3\mathbf{i} + 5\mathbf{j}$.

6.3 Polar Coordinates

* Many times it is much easier to graph on a polar coordinate plane rather than rectangular coordinate plane.

*

A point P on the polar coordinate = (r, θ) .

- R is a directed distance from the pole to P (can be pos or neg)
- θ is an angle from the polar axis to the line segment from the pole to P (can be degrees or radians)
 - Positive angles are measured counterclockwise
 - Negative angles are measured clockwise

+ talk about $r = 3$ & $\theta = \pi/4$

Relations between polar and rectangular coordinates
(conversions)

$$x = r \cos\theta$$

$$r = \sqrt{x^2 + y^2}$$

$$r^2 = x^2 + y^2$$

$$y = r \sin\theta$$

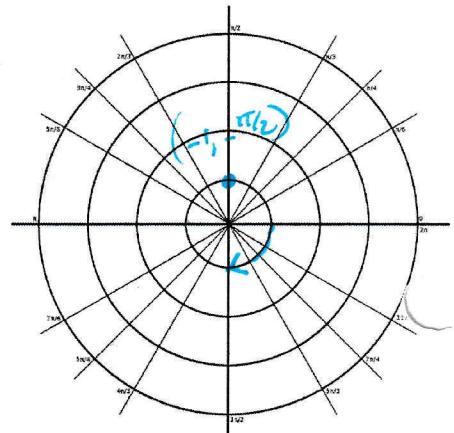
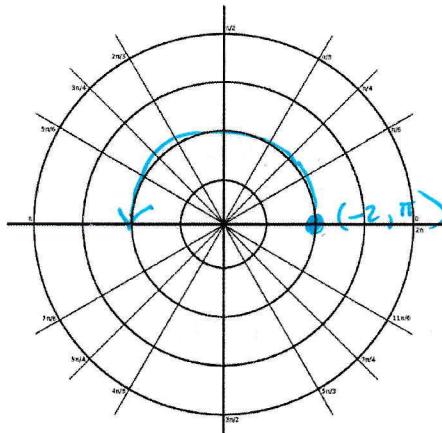
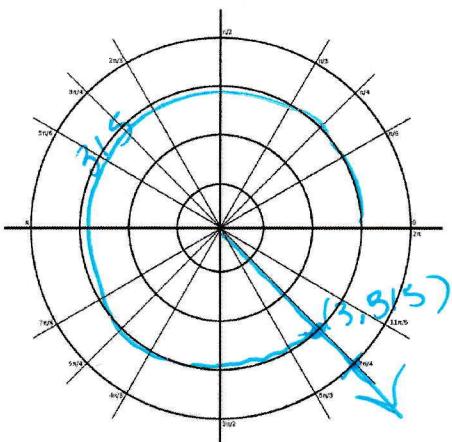
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

1) Plot the following polar coordinates:

a. $(3, 315^\circ)$

b. $(-2, \pi)$

c. $(-1, -\frac{\pi}{2})$



$r \theta$

2) Find another representation of $(5, \frac{\pi}{4})$ in which:

r is positive and $2\pi < \theta < 4\pi$

$$+2\pi$$

$$(5, \frac{9\pi}{4})$$

add 2π & don't
change r

r is negative and $0 < \theta < 2\pi$

* add π & replace r by $-r$

$$(-5, \frac{5\pi}{4})$$

add π & change
 r with $-r$

r is positive and $-2\pi < \theta < 0$

Subtract 2π & do not change r

$$(5, -\frac{7\pi}{4})$$

3) Find the rectangular coordinates of the points with the following polar coordinates:

$$(r, \theta) \quad a. (3, \pi)$$

$$\begin{aligned} x = r \cos \theta &= 3 \cos \pi \\ &= 3(-1) \\ &= -3 \end{aligned}$$

$$\begin{aligned} y = r \sin \theta &= 3 \sin \pi \\ &= 3(0) \\ &= 0 \end{aligned}$$

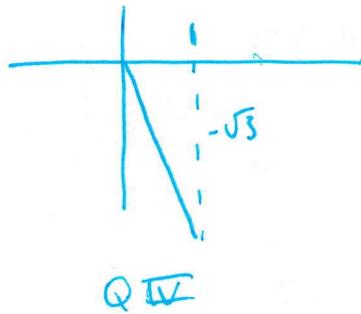
$$(-3, 0)$$

$$b. (-10, \frac{\pi}{6})$$

$$\begin{aligned} x &= -10 \cos \frac{\pi}{6} & y &= -10 \sin \frac{\pi}{6} \\ &= -10(\frac{\sqrt{3}}{2}) & &= -10(\frac{1}{2}) \end{aligned}$$

$$(-5\sqrt{3}, -5)$$

4) Find polar coordinates of the point whose rectangular coordinates are $(1, -\sqrt{3})$.



$$\begin{aligned} r &= \sqrt{1^2 + (-\sqrt{3})^2} \\ &= \sqrt{1 + 3} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

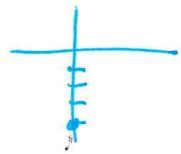
$$\tan \theta = \frac{-\sqrt{3}}{1}$$

$$\tan \theta = -\sqrt{3}$$

$$\theta = \frac{5\pi}{3}$$

$$(2, \frac{5\pi}{3})$$

- 5) Find polar coordinates of the point whose rectangular coordinates are $(0, -4)$. Express θ in radians.



$$r = \sqrt{0^2 + 4^2} \\ = 4$$

$$\tan \theta = -\frac{4}{0}$$

undefined

$$\theta = \frac{3\pi}{2}$$

$$\boxed{(4, \frac{3\pi}{2})}$$

- 6) Convert each rectangular equation to a polar equation that expresses r in terms of θ :

a. $3x - y = 6$

$$3r\cos\theta - r\sin\theta = 6$$

$$r(3\cos\theta - \sin\theta) = 6$$

$$r = \frac{6}{3\cos\theta - \sin\theta}$$

standard form for circle
 $r=1$ center $(0, -1)$

b. $x^2 + (y + 1)^2 = 1$

$$(r\cos\theta)^2 + (r\sin\theta + 1)^2 = 1$$

$$r^2\cos^2\theta + r^2\sin^2\theta + 2r\sin\theta + 1 = 1$$

$$r^2(\cos^2\theta + \sin^2\theta) + 2r\sin\theta = 0$$

$$r^2 + 2r\sin\theta = 0$$

$$r(r + 2\sin\theta) = 0$$

$$\boxed{(r=0 \text{ or } r = -2\sin\theta)}$$

Single point
(the pole)

$$\boxed{r = -2\sin\theta}$$

Polar \rightarrow Rectangular $x \& y$ instead of $r \& \theta$

$$r^2 = x^2 + y^2 \quad r\cos\theta = x \quad r\sin\theta = y \quad \tan\theta = \frac{y}{x}$$

7) Convert each polar equation to a rectangular equation in x and y .

a. $r = 4$

$$r^2 = 16$$

$$\boxed{x^2 + y^2 = 16}$$

b. $\theta = \frac{3\pi}{4}$

$$\tan\theta = \frac{y}{x}$$

$$\tan\theta = \tan\frac{3\pi}{4} \quad (\text{take tan of each side})$$

$$\tan\theta = -1$$

$$\frac{y}{x} = -1$$

$$\boxed{y = -x}$$

c. $r = -2\sec\theta$

$$r\cos\theta = x \quad \sec = \frac{1}{\cos}$$

$$r = \frac{-2}{\cos\theta}$$

$$r\cos\theta = -2$$

$$\boxed{x = -2}$$

d. $r = 10\sin\theta$

mult both sides by r

$$r^2 = 10r\sin\theta$$

$$\boxed{x^2 + (y-5)^2 = 25}$$

$$x^2 + y^2 = 10y$$

$$x^2 + y^2 - 10y = 0$$

$$x^2 + y^2 - 10y + 25 = 25$$

complete square

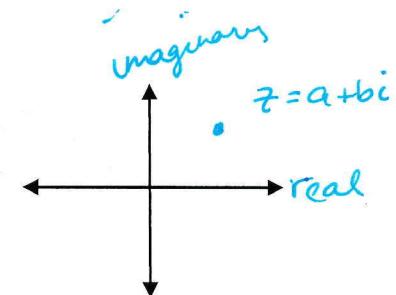
6.4 Graphs of Polar Coordinates

*Handout

6.5 Complex Numbers in Polar Form

Complex Form: $a + bi$ (rectangular form)

real *imaginary*



The absolute value of a complex number:

*Distance from the origin to the point z in the complex plane

$$|z| = |a + bi| = \sqrt{a^2 + b^2} = r$$

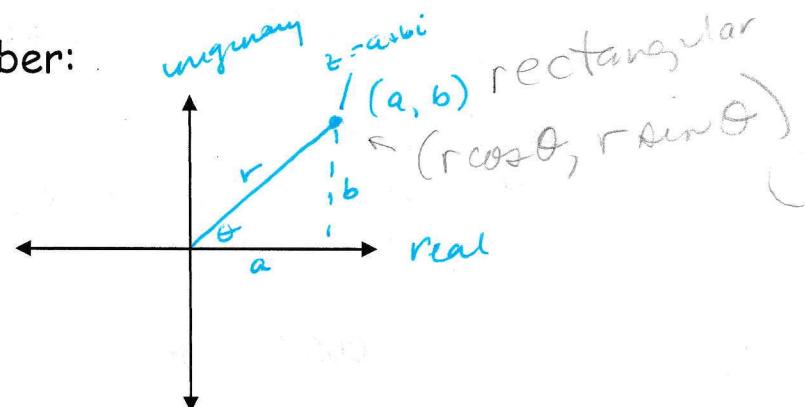
Polar Form of a complex number:

$$r = \sqrt{a^2 + b^2}$$

$$a = r \cos \theta$$

$$b = r \sin \theta$$

~~$c = r \tan \theta$~~



$$z = \underbrace{r \cos \theta}_{a} + \underbrace{(r \sin \theta)i}_{b}$$

or

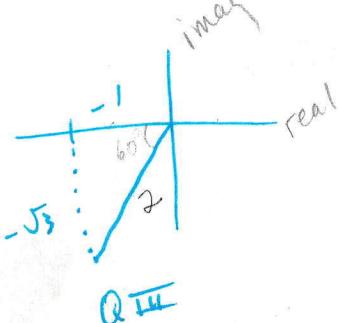
$$z = r(\cos \theta + i \sin \theta)$$

$r \text{ cis } \theta$

$$a \quad bi$$

1) Plot $z = -1 - i\sqrt{3}$ in the complex plane. Then write z in polar form. Express the argument in radians.

$$\begin{aligned} a &= -1 \\ b &= -\sqrt{3} \end{aligned}$$



$$\begin{aligned} r &= \sqrt{(-1)^2 + (-\sqrt{3})^2} \\ &= \sqrt{1+3} \\ &= 2 \end{aligned}$$

$$\tan \theta = \frac{-\sqrt{3}}{-1}$$

$$= \sqrt{3}$$

$4\pi/3$

$$\begin{aligned} z &= r(\cos \theta + i \sin \theta) \\ z &= 2(\cos 4\pi/3 + i \sin 4\pi/3) \\ &= 2 \left(\cos \left(\frac{4\pi}{3} \right) + i \sin \left(\frac{4\pi}{3} \right) \right) \\ &= 2 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right) \end{aligned}$$

2) Write $z = 4(\cos 30^\circ + i \sin 30^\circ)$ in rectangular form.

$$= 4 \left(\frac{\sqrt{3}}{2} + i \left(\frac{1}{2} \right) \right)$$
$$\boxed{z = 2\sqrt{3} + 2i}$$
$$z = a + bi$$

Product of Two Complex Numbers in Polar Form:

-Product/Multiply

Multiply the radii and add the θ 's

-Divide/Quotient

Divide the radii and subtract the θ 's

3) Find the product of $z_1 = 6(\cos 40^\circ + i \sin 40^\circ)$ and $z_2 = 5(\cos 20^\circ + i \sin 20^\circ)$. Leave the answer in polar form.

$$[6(\cos 40^\circ + i \sin 40^\circ)][5(\cos 20^\circ + i \sin 20^\circ)]$$

$$(6 \cdot 5)[\cos(40+20) + i \sin(40+20)]$$

$$= \boxed{30(\cos 60^\circ + i \sin 60^\circ)} = 30 \text{ cis } 60^\circ$$

OR

$$= 30 \text{ cis } \frac{\pi}{3}$$

4) Find the quotient of $z_1 = 50(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})$ and

$$z_2 = 5(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$$

$$\frac{50}{5} \frac{(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})}{(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})}$$

$$= 10 \cos \left(\frac{4\pi}{3} - \frac{\pi}{3} \right) + i \sin \left(\frac{4\pi}{3} - \frac{\pi}{3} \right)$$

$$10 \text{ cis } \pi$$

$$= \boxed{10 \cos \pi + i \sin \pi}$$

DeMoivre's Theorem

Let $z = r(\cos\theta + i \sin\theta)$ be a complex number in polar form. If n is a positive integer, then z to the n th power is:

$$z^n = [r(\cos\theta + i \sin\theta)]^n = r^n (\cos\theta + i \sin\theta)^n$$

5) Find $[2(\cos 30^\circ + i \sin 30^\circ)]^5$

$$= 2^5 [\cos(5 \cdot 30^\circ) + i \sin(5 \cdot 30^\circ)]$$

$$= 32 \cos 150^\circ + i \sin 150^\circ$$

$$= 32 \left(-\frac{\sqrt{3}}{2} + i \frac{1}{2}\right)$$

$$= \boxed{-16\sqrt{3} + 16i}$$

$$32 \text{ cis } 150^\circ$$

$$\text{or } 32 \text{ cis } \frac{5\pi}{6}$$

6) Find $(1+i)^4$ using DeMoivre's Theorem. Write the answer in rectangular form.

$$r = \sqrt{a^2+b^2}$$

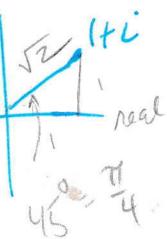
$$= \sqrt{1^2+1^2}$$
$$= \sqrt{2}$$

$$\tan \theta = \frac{1}{1}$$

$$\tan \theta = 1$$

$$\frac{\pi}{4}$$

$$(r, \theta) = \left(\sqrt{2}, \frac{\pi}{4}\right)$$



$$1+i = r(\cos\theta + i \sin\theta) = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) \sqrt{2} \text{ cis } \frac{\pi}{4}$$

$$(1+i)^4 = (\sqrt{2})^4 [\cos(4 \cdot \frac{\pi}{4}) + i \sin(4 \cdot \frac{\pi}{4})]$$

$$= 4 (\cos 2\pi + i \sin 2\pi)$$

$$= 4 (-1 + 0i)$$

$$= \boxed{-4} \text{ or } \boxed{-4+0i}$$

$$4 \text{ cis } 2\pi$$